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Vacuum Structure and Chiral Symmetry Breaking

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Abstract

We relate here the vacuum structure of chiral symmetry breaking to a wide class of low energy hadronic properties to examine consistency with experimental results. Phase transition of chiral symmetry breaking is described through explicit vacuum realignment. We find that for the present ansatz, vacuum structure of the light quark sector can be determined from $f_\pi = 92$ MeV, which then yields that $\Gamma(\pi^0 \rightarrow 2\gamma) = 8.3$ eV and, $R_{ch} = 0.67$ fms, in agreement with experiments. This amounts to three independent 'observations' of the vacuum structure of the light quark sector. For the calculation of $\Gamma(\pi^0 \rightarrow 2\gamma)$, we do not use the anomaly equation - local gauge invariance for vacuum realignment becomes adequate. The above width is sensitive to the vacuum structure, and can give evidence regarding progress towards chiral symmetry restoration in QGP. The corresponding results for s quark sector are moderately good. We also exploit similar ideas for heavy quarks, but here the experimental data are rare, and the theoretical techniques are likely to need refinements.

I. INTRODUCTION

It is now accepted that quantum chromodynamics (QCD) is the correct theory for strong interaction physics of quarks and gluons, and, at a secondary level, of hadrons. However, at present no reliable method is known for the solution of such problems. The basic difficulty appears to be an understanding of the ground state properties of QCD or its vacuum structure, which plays an important role [1], and has been under intense investigation since a long time dealing with the nontrivial vacuum structure for gluons [2–4]. A parallel non-perturbative approach for quark condensates has also been made through chiral symmetry breaking with pion as a Goldstone mode [5]. We have looked at the same in the context of potentials [6] as well as for Nambu Jona Lasinio model [7] with symmetry breaking as vacuum realignment [8].

Our approach to these problems [4,6,7] mainly has been variational, where the parameters or functions are determined through minimisation of energy density. At the present level of sophistication, determination of the vacuum structure through such a minimisation does not seem to be very reliable. In view of this we had also considered an alternative approach [9] where on intuitive grounds we take a simple ansatz for the vacuum structure, and then examine the consequences for observed hadronic properties. This appeared to describe a host of phenomena as being related to the vacuum structure for chiral symmetry breaking [9]. Here we shall generalise the same to discuss a wider class of problems to determine the vacuum structure from experimental observations.

The paper is organised as follows. In section **II** we discuss an ansatz for the vacuum structure for chiral symmetry breaking to see whether *post facto* it can be the correct description for vacuum realignment. In particular we obtain the four component Dirac spinors as related to the vacuum structure. In section **III** we obtain the pion and kaon wave functions in terms of vacuum realignment of u, d and s quarks. We then use the experimental values of f_π and f_K *to determine* the vacuum realignment of the above quarks for the present ansatz. In section **IV** we consider the process $\pi^0 \to 2\gamma$ as a consequence of vacuum realignment

with local gauge invariance. This yields that $\Gamma(\pi^0 \rightarrow 2\gamma) = 8.3$ eV *without* fixing any additional parameter or using the anomaly equation. In section **V** we discuss the masses of the pseudoscalar mesons through approximate chiral symmetry breaking in terms of the 'small' Lagrangian masses of the quarks. We then use the same to determine the vacuum structure for c-quark and b-quark condensates, and obtain their decay constants along with some additional results. In section **VI** we obtain the charge radii of the pion and the kaon where the mesons in motion are defined through Lorentz boosting of the corresponding states at rest. In section **VII** we summarise the results, mention the limitations of the present paper and discuss the scope for further work. We also discuss here the relevance of the present ideas in quark gluon plasma for chiral symmetry restoration.

The method considered here is a non-perturbative one as we use only equal time quantum algebra but is limited by the choice of ansatz functions in the calculations. The techniques have been applied earlier to solvable cases to examine its reliability [10] as well as to physically more relevant ground state structures in high energy physics [4,11] and nuclear physics [12,13]. We examine here chiral symmetry breaking with an ansatz, and then relate the same to low energy hadronic properties in an attempt to identify vacuum structure through experimental results.

II. CHIRAL SYMMETRY BREAKING FOR QUARKS

For the consideration of chiral symmetry breaking, we shall take the perturbative vacuum state with chiral symmetry as $|vac>$. In this basis quarks are massless, or have a small mass. We shall then assume a specific vacuum realignment which breaks chiral symmetry. As stated we shall then relate the vacuum structure with experimental observations [9].

We have seen earlier [6–9] that chiral symmetry breaking takes place with the formation of quark antiquark condensates in the perturbative vacuum. We shall thus take the destabilised vacuum, $|vac'>$ as

$$|vac'>= U_Q|vac> \tag{2.1}$$

where U_Q is given as

$$U_Q = e^{B_0^\dagger - B_0} \tag{2.2}$$

with

$$B_0^\dagger = \int q_I^{0i}(\mathbf{k})^\dagger h_i(\mathbf{k})(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\tilde{q}_I^{0i}(-\mathbf{k})d\mathbf{k}. \tag{2.3}$$

In the above, i stands for the flavour of the quark with $i = 1, 2, 3, \cdots$ standing for $u, d, s, \cdots$ quarks. The subscript I and the superscript '0' indicate that these are two component quark or antiquark operators which create or annihilate quanta of the perturbative or chiral vacuum. We are accepting the fact that there may be condensates also for heavy quarks. $h^i(\mathbf{k})$ are the ansatz functions which describe vacuum realignment for quark of flavour i. We shall take a simple form for them on intuitive grounds [9] and then relate the same to observations. The colour quantum number of quarks has been suppressed. We are taking the quarks after a Kobayashi Maskawa rotation [14], so that the Lagrangian mass matrix for them is diagonal. Our ansatz here has an obvious parallel with superconductivity [5,15].

The above two component creation or annihilation operators arise in the momentum space expansion of the four component field operators corresponding to the perturbative basis. They are given as [6–9]

$$\begin{aligned}\psi(\mathbf{x}) &\equiv \frac{1}{(2\pi)^{3/2}}\int \tilde{\psi}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}d\mathbf{k} \\ &= \frac{1}{(2\pi)^{3/2}}\int \left[U_0(\mathbf{k})q_I^0(\mathbf{k}) + V_0(-\mathbf{k})\tilde{q}_I^0(-\mathbf{k})\right]e^{i\mathbf{k}\cdot\mathbf{x}}d\mathbf{k},\end{aligned} \tag{2.4}$$

where [16,17]

$$\begin{aligned}U_0(\mathbf{k}) &= \begin{pmatrix} cos\frac{\chi_0(\mathbf{k})}{2} \\ \sigma\cdot\hat{k}sin\frac{\chi_0(\mathbf{k})}{2}\end{pmatrix}, \\ V_0(-\mathbf{k}) &= \begin{pmatrix} -\sigma\cdot\hat{k}sin\frac{\chi_0(\mathbf{k})}{2} \\ cos\frac{\chi_0(\mathbf{k})}{2}\end{pmatrix}.\end{aligned} \tag{2.5}$$

The above form is so taken that it satisfies the equal time algebra [16,17], and we have suppressed both colour and flavour indices. In the above, for *free* quark fields of mass m_Q, we have [16]

$$cos\frac{\chi_0(\mathbf{k})}{2}=\Big(\frac{p^0+m_Q}{2p^0}\Big)^{\frac{1}{2}},\qquad sin\frac{\chi_0(\mathbf{k})}{2}=\Big(\frac{p^0-m_Q}{2p^0}\Big)^{1/2},\tag{2.6}$$

with $p^0=(m_Q^2+\mathbf{k}^2)^{\frac{1}{2}}$. We may further note that *when chiral symmetry is unbroken and thus the quarks are massless*, $\chi_0(\mathbf{k})=\pm\pi/2$. As we shall now see, this can be used in an essential manner to obtain the four component quark field operators when chiral symmetry gets broken.

Now, for any operator O in the $|vac>$ basis, the corresponding operator O' in the $|vac'>$ basis is given through

$$O'=U_QOU_Q^\dagger\tag{2.7}$$

which allows us to write the creation and annihilation operators in the $|vac'\rangle$ basis. We may here first note from equation (2.7) that *the operator U_Q remains unchanged in form when we go from one basis to the other*, which follows from the fact that $U_Q'=U_QU_QU_Q^\dagger=U_Q$. We can thus take the operator U_Q as defined by equations (2.2) and (2.3) in $|vac\rangle$ basis, or in $|vac'\rangle$ basis.

Now, the creation and annihilation operators in the chiral symmetry broken basis are given through the inverse of equation (2.7) as

$$\begin{aligned}\begin{pmatrix}q_I^0(\mathbf{k})\\ \tilde{q}_I^0(-\mathbf{k})\end{pmatrix}&=U_Q^\dagger\begin{pmatrix}q_I(\mathbf{k})\\ \tilde{q}_I(-\mathbf{k})\end{pmatrix}U_Q\\ &=\begin{pmatrix}cos(h(\mathbf{k})) & \sigma\cdot\hat{k}sin(h(\mathbf{k}))\\ -\sigma\cdot\hat{k}sin(h(\mathbf{k})) & cos(h(\mathbf{k}))\end{pmatrix}\begin{pmatrix}q_I(\mathbf{k})\\ \tilde{q}_I(-\mathbf{k})\end{pmatrix}.\end{aligned}\tag{2.8}$$

In the above we have taken U_Q in the $|vac'\rangle$ basis, with $q_I(\mathbf{k})$ and $\tilde{q}_I(-\mathbf{k})$ also being the two-component quark annihilation and antiquark creation operators in $|vac'\rangle$. As earlier we have suppressed the spin, flavour and colour indices. From equations (2.4) and (2.8) we relate the quark field expansions in the two basis through the equation

$$\tilde{\psi}(\mathbf{k})=U_0(\mathbf{k})q_I^0(\mathbf{k})+V_0(-\mathbf{k})\tilde{q}_I^0(-\mathbf{k})=U(\mathbf{k})q_I(\mathbf{k})+V(-\mathbf{k})\tilde{q}_I(-\mathbf{k})\tag{2.9}$$

so that the four component spinors after chiral symmetry breaking become known. We then easily obtain from equation (2.8) that the spinors $U(\mathbf{k})$ and $V(\mathbf{k})$ after chiral symmetry breaking are given as

$$U(\mathbf{k}) = \begin{pmatrix} cos\frac{\chi(\mathbf{k})}{2} \\ \sigma \cdot \hat{k} sin\frac{\chi(\mathbf{k})}{2} \end{pmatrix},$$

$$V(-\mathbf{k}) = \begin{pmatrix} -\sigma \cdot \hat{k} sin\frac{\chi(\mathbf{k})}{2} \\ cos\frac{\chi(\mathbf{k})}{2} \end{pmatrix}, \tag{2.10}$$

where,

$$\frac{\chi(\mathbf{k})}{2} = \frac{\chi_0}{2} - h(\mathbf{k}). \tag{2.11}$$

Now, when we know χ_0 *we also know* $\chi(\mathbf{k})$ *in terms of* $h(\mathbf{k})$ *for the vacuum structure.* For definiteness we take for the chiral vacuum $\chi_0 = \pi/2$. *Then the vacuum structure explicitly gives also the four component quark field operators* [9]. In what follows we shall exploit this result. We also note that $f_i(\mathbf{k})$ and $g_i(\mathbf{k})$ in Ref. [16] for quarks of flavour i here correspond to

$$f_i(\mathbf{k}) = cos\left(\frac{1}{2}\chi_i(\mathbf{k})\right); \qquad |\mathbf{k}|g_i(\mathbf{k}) = sin\left(\frac{1}{2}\chi_i(\mathbf{k})\right). \tag{2.12}$$

We further note that

$$< vac'|\bar{\psi}_i(\mathbf{x})\psi_i(\mathbf{y})|vac' >= -\frac{6}{(2\pi)^3}\int cos\chi_i(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} d\mathbf{k}, \tag{2.13}$$

where, as earlier i is the flavour of the quark, and, the factor 6 arises from colour and spin degress of freedom. We note that in the above $cos\chi_i(\mathbf{k}) = sin(2h_i(\mathbf{k}))$, describing the above function in terms of the quark antiquark correlations for vacuum realignment.

We now make our intuitive approximation about the vacuum structure on the basis of the above results. *We shall approximate the function in equation (2.13) by a Gaussian.* Thus, with $cos\chi_i(\mathbf{0}) = 1$ in equation (2.10), we write

$$sin(2h_i(\mathbf{k})) = cos(\chi_i(\mathbf{k})) = e^{-\frac{1}{2}R_i^2\mathbf{k}^2}, \tag{2.14}$$

with a single dimensional parameter R_i for each quark of flavour i. This simple form appears to be consistent with some low energy hadronic phenomenology as earlier [9] and we shall take the same as a "zeroeth order" ansatz.

We shall further assume that initially we have $SU(2)_L \times SU(2)_R$ chiral symmetry which breaks to the custodial symmetry $SU(2)_V$. We then have $R = R_1 = R_2$ for u, d quarks.

III. VACUUM STRUCTURE AND PION AND KAON DECAY CONSTANTS

In the present section we shall first use Goldstone theorem to identify the quark antiquark wave functions for the π^+ and K^+ states as related to the vacuum structure [6,7,9], and then link the same to the corresponding decay constants. For the sake of completeness, we briefly recall [9] the calculations for pion here.

We note that the chiral generator corresponding to the π^+ state is given as

$$Q_5^{\pi^+} = \int \psi_1(\mathbf{x})^\dagger \gamma_5 \psi_2(\mathbf{x}) d\mathbf{x}, \tag{3.1}$$

with ψ_1 and ψ_2 standing for u and d quarks. Clearly when chiral symmetry is there, $Q_5^{\pi^+}|vac\rangle = 0$, whereas when chiral symmetry is broken, $Q_5^{\pi^+}|vac'\rangle \neq 0$, and will correspond to the π^+ state of zero momentum [6–9]. Hence on using equation (3.1), through a direct evaluation we identify the π^+ state of zero momentum as, with $cos\chi(\mathbf{k}) = sin(2h(\mathbf{k}))$,

$$| \pi^+(\mathbf{0}) >= N_\pi \cdot \frac{1}{\sqrt{6}} \int u_I(\mathbf{k})^\dagger \tilde{d}_I(-\mathbf{k}) cos(\chi(\mathbf{k})) d\mathbf{k} \mid vac' >, \tag{3.2}$$

where, N_π is a normalisation constant. For the u and d quarks we have $R_1 = R_2 = R$, and thus $\chi_1(\mathbf{k}) = \chi_2(\mathbf{k}) \equiv \chi(\mathbf{k})$. The colour and spin indices of quarks and antiquarks have been suppressed. The normalisation constant N_π is given through the equation [16]

$${N_\pi}^2 \int cos^2(\chi(\mathbf{k})) d\mathbf{k} = 1. \tag{3.3}$$

For the present ansatz of equation (2.14) we then have from equation (3.2) the pion wave function $\tilde{u}_\pi(\mathbf{k})$ as [16]

$$\tilde{u}_\pi(\mathbf{k}) = N_\pi cos(\chi(\mathbf{k})) = N_\pi \exp^{-\frac{1}{2}R^2\mathbf{k}^2}, \tag{3.4}$$

where, on integration we have

$$N_\pi = \frac{R_\pi^{3/2}}{\pi^{3/4}} = 0.424 \times R_\pi^{3/2}. \tag{3.5}$$

Clearly the state as in equation (3.2) as the Goldstone mode will be accurate to the extent we determine the vacuum structure sufficiently accurately through variational or any other method so that $|vac' >$ is an eigenstate of the total Hamiltonian. In general, if Q_a is a generator for a symmetry breaking direction of a global symmetry [18], then

$$Q_a|vac\rangle = 0 \tag{3.6}$$

whereas

$$Q_a|vac'\rangle \neq 0, \tag{3.7}$$

and, it defines the state for the Goldstone mode [6–9]. This result in particular yields wave function of the pion from the vacuum structure for any example of chiral symmetry breaking, and is the new feature of looking at phase transition through vacuum realignment [8].

We had earlier considered low energy hadronic properties [16] with an assumed form of four component quark field operators. This is equivalent to a choice of $cos(\chi(\mathbf{k}))$. In addition, we had taken Gaussian wave functions for mesons and then discussed the phenomenology. Now from the vacuum structure the pion wave function is known and, the four component quark field operators are also known. This decreases the number of independent quantities making the earlier model [16] more restrictive, giving rise to predictions.

We next use the pion decay constant f_π to determine the vacuum structure of chiral symmetry breaking for u and d quarks [9]. The decay constant f_π as calculated earlier in terms of the wave function including here the relativistic corrections [16,19] is given through

$$\left|(1 + 2g_1^2\bigtriangledown^2)u_\pi(\mathbf{0})\right| = \frac{f_\pi(m_\pi)^{1/2}}{\sqrt{6}}, \tag{3.8}$$

where the value of the wave function is taken at the space origin, and, g_1 is a differentiation operator in coordinate space given through equation (2.12) with $g_1 = g_2$ for u and d quarks. On taking the Fourier transform, the left hand side of the above equation becomes

$$\frac{1}{(2\pi)^{3/2}} \int cos\chi(\mathbf{k}) u_\pi(\mathbf{k}) d\mathbf{k},$$

where we have substituted that $1 - 2g_1(\mathbf{k})^2\mathbf{k}^2 = cos(\chi(\mathbf{k}))$. Hence from equations (3.3), (3.4) and (3.8) we obtain that

$$\frac{1}{(2\pi)^{3/2}} \cdot \frac{1}{N_\pi} = \frac{f_\pi (m_\pi)^{1/2}}{\sqrt{6}}, \tag{3.9}$$

so that, with $f_\pi = 92$ MeV, we get

$$N_\pi = 4.534 \text{ GeV}^{-3/2}. \tag{3.10}$$

Hence, from equation (3.5) we obtain that, with $R_1 = R_2$,

$$R_1^2 = R_2^2 = \pi \times N_\pi^{4/3} = 23.58 \text{ GeV}^{-2}. \tag{3.11}$$

This (i) determines the vacuum structure in the light quark sector as in equation (2.14), (ii) yields the pion wave function as in equation (3.4), and, (iii) gives the four component quark field operators for light quarks through eqaution (2.10).

We next utilise the value of f_K to determine the vacuum structure for the s-quark. The generator $Q_5^{K^+}$ for K^+ is given as, with ψ_3 standing for the s-quark,

$$Q_5^{K^+} = \int \psi_1(\mathbf{x})^\dagger \gamma_5 \psi_3(\mathbf{x}) d\mathbf{x}. \tag{3.12}$$

Hence on direct evaluation we identify the K^+ state of zero momentum as

$$\begin{aligned} \mid K^+(\mathbf{0}) > = N_K \cdot \frac{1}{\sqrt{6}} \int u_I(\mathbf{k})^\dagger \tilde{s}_I(-\mathbf{k}) \Big[& cos(\frac{1}{2}\chi_1(\mathbf{k})) cos(\frac{1}{2}\chi_3(\mathbf{k})) \\ & - sin(\frac{1}{2}\chi_1(\mathbf{k})) sin(\frac{1}{2}\chi_3(\mathbf{k})) \Big] d\mathbf{k} \mid vac' >, \end{aligned} \tag{3.13}$$

where, N_K is a normalisation constant. The wave function for kaon $\tilde{u}_K(\mathbf{k})$ thus is given as

$$\tilde{u}_K(\mathbf{k}) = N_K \Big[cos(\frac{1}{2}\chi_1(\mathbf{k})) cos(\frac{1}{2}\chi_3(\mathbf{k})) - sin(\frac{1}{2}\chi_1(\mathbf{k})) sin(\frac{1}{2}\chi_3(\mathbf{k})) \Big], \tag{3.14}$$

with the normalisation constant N_K determined through

$$N_K^2 \int \Big[cos(\frac{1}{2}\chi_1(\mathbf{k}))cos(\frac{1}{2}\chi_3(\mathbf{k})) - sin(\frac{1}{2}\chi_1(\mathbf{k}))sin(\frac{1}{2}\chi_3(\mathbf{k}))\Big]^2 d\mathbf{k} = 1, \tag{3.15}$$

which gives

$$\frac{1}{N_K^2} = 2\pi \int \Big[1 - sin(\chi_1(|\mathbf{k}|))sin(\chi_3(|\mathbf{k}|)) + cos(\chi_1(|\mathbf{k}|))cos(\chi_3(|\mathbf{k}|))\Big]|\mathbf{k}|^2 d|\mathbf{k}|. \tag{3.16}$$

Parallel to equation (3.8), the Van Royen Weisskopf relation for K^+ meson in momentum space is *now calculated as* [16]

$$\frac{1}{(2\pi)^{3/2}}\Big|\int [cos(\frac{1}{2}\chi_1(\mathbf{k}))cos(\frac{1}{2}\chi_3(\mathbf{k})) - sin(\frac{1}{2}\chi_1(\mathbf{k}))sin(\frac{1}{2}\chi_3(\mathbf{k}))]u_K(\mathbf{k})d\mathbf{k}\Big| = \frac{f_K(m_K)^{1/2}}{\sqrt{6}}, \tag{3.17}$$

which from equations (3.14) and (3.15) is equivalent to

$$\frac{1}{(2\pi)^{3/2}} \cdot \frac{1}{N_K} = \frac{f_K(m_K)^{1/2}}{\sqrt{6}}. \tag{3.18}$$

With $f_K = 1.2\ f_\pi$, this yields that

$$N_K = 1.9935\ \text{GeV}^{-3/2}. \tag{3.19}$$

Now, in equation (3.15) R_1 is known as in equation (3.11). Hence with equation (3.19) we can obtain the value of $R_3 = R_s$. Through a numerical evaluation we then obtain that

$$R_s^2 = 4.084\ \text{GeV}^{-2}. \tag{3.20}$$

This determines the vacuum structure for the s-quark, the four component spinors for the s-quark, as well as the kaon wave function in the limit of exact chiral symmetry breaking.

We can now obtain also the values of the u, d and s quark antiquark condensates. In fact, from equations (2.13) and (2.14) we easily obtain that

$$-\langle vac'|\bar{\psi}_i(\mathbf{x})\psi_i(\mathbf{x})|vac'\rangle = \frac{3}{\sqrt{2}} \cdot \frac{1}{\pi\sqrt{\pi}} \cdot \frac{1}{R_i^3}, \tag{3.21}$$

so that we have

$$\left(-\langle\bar{u}u\rangle\right)^{1/3} = 149 \text{ MeV}; \quad \left(-\langle\bar{u}u\rangle - \langle\bar{d}d\rangle\right)^{1/3} = 188 \text{ MeV}; \quad \left(-\langle\bar{s}s\rangle\right)^{1/3} = 359 \text{ MeV}. \tag{3.22}$$

The condensate expressions for u and d quarks are smaller than usual. Chiral symmetry is restored when $R_i \to \infty$, and, the condensates as above vanish.

IV. MINIMAL ELECTROMAGNETIC COUPLING AND $\pi^0 \to 2\gamma$ DECAY

In this section we shall see that the vacuum structure in the light quark sector along with local gauge invariance of $|vac'\rangle$ will be adequate to yield the π^0 width. For this purpose we shall obtain minimal coupling as explained below, and then calculate $\Gamma(\pi^0 \to 2\gamma)$. We shall see that this yields the correct experimental value of the same *without* additional parameters, and without using the anomaly equation.

In order to include electromagnetic interactions correctly, we should consider vacuum realignment as in equations (2.2) and (2.3) so that gauge invariance is maintained while restructuring of vacuum takes place. Let us consider the covariant derivative of the quark field $\psi_q(\mathbf{x})$ given as

$$D_i\psi_q(\mathbf{x}) \equiv \left(-i\partial_i - e_q A_i(\mathbf{x})\right)\psi_q(\mathbf{x}). \tag{4.1}$$

We shall find it convenient to write the above in the Fourier transform space. With

$$A_i(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int \tilde{A}_i(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{k}, \tag{4.2}$$

we can write the Fourier transform of covariant derivative as

$$\frac{1}{(2\pi)^{3/2}} \int D_i\psi_q(\mathbf{x}) \exp^{-i\mathbf{k}\cdot\mathbf{x}} d\mathbf{x} = \int d\mathbf{k}'[\mathbf{k}_i\delta(\mathbf{k}-\mathbf{k}') - \frac{e_q}{(2\pi)^{3/2}}\tilde{A}_i(\mathbf{k}-\mathbf{k}')]\tilde{\psi}_q(\mathbf{k}'). \tag{4.3}$$

We now define the continuous matrix

$$\mathbf{K}_i(\mathbf{k},\mathbf{k}') \equiv \mathbf{k}_i\delta(\mathbf{k}-\mathbf{k}') - \frac{e_q}{(2\pi)^{3/2}}\tilde{A}_i(\mathbf{k}-\mathbf{k}'). \tag{4.4}$$

The usual covariant derivative for minimal coupling in momentum space thus consists of the substitution

$$\mathbf{k}_i\tilde{\psi}_q(\mathbf{k}) \to \int \mathbf{K}_i(\mathbf{k},\mathbf{k}')\tilde{\psi}_q(\mathbf{k}')d\mathbf{k}'. \tag{4.5}$$

Let us however note that

$$\tilde{\psi}_q(\mathbf{k}) = U(\mathbf{k})q_I(\mathbf{k}) + V(-\mathbf{k})\tilde{q}_I(-\mathbf{k}). \tag{4.6}$$

Let us generalise the above with minimal coupling for *two component* operators $q_I(\mathbf{k})$ and $\tilde{q}_I(-\mathbf{k})$ as proposed earlier [16]. Then the corresponding Fourier transform of the Dirac field is given as

$$\tilde{\psi}_q^{minml}(\mathbf{k}) = U(\mathbf{K})q_I(\mathbf{k}) + V(-\mathbf{K})\tilde{q}_I(-\mathbf{k}), \tag{4.7}$$

where the superscript *minml* stands for including two component minimal electromagnetic coupling, and, $\mathbf{K}$ is a matrix as in equation (4.4). The difference between equation (4.5) and (4.7) is that in the later case the spinors themselves shall contain the covariant derivatives.

We shall now show that the above result follows due to vacuum realignment when $|vac'\rangle$ remains gauge invariant. To keep $|vac'\rangle$ gauge invariant, we replace equation (2.3) by

$$B_0^\dagger = \int q_I^{0i}(\mathbf{k})^\dagger (h_i(\boldsymbol{\sigma}\cdot\mathbf{K}))(\mathbf{k},\mathbf{k}')\tilde{q}_I^{0i}(-\mathbf{k}')d\mathbf{k}d\mathbf{k}'. \tag{4.8}$$

In the above we have replaced $\mathbf{k}^2$ by the matrix $(\boldsymbol{\sigma}\cdot\mathbf{K})^2$ *everywhere* for minimal coupling, since in Dirac equation the square of momentum always arises through $(\boldsymbol{\sigma}\cdot\mathbf{k})^2 = \mathbf{k}^2$, and, have replaced $h_i(\mathbf{k})(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})$ by the matrix $h_i(\boldsymbol{\sigma}\cdot\mathbf{K})$. Clearly equation (4.8) maintains gauge invariance of $|vac'\rangle$ at two component level, and, for massless quarks, gauge invariance at two compenent level and at usual four component level are equivalent since here the spinors are constants. Now the Bogoliubov transformations (2.8) contain covariant derivatives, and thus in $|vac'\rangle$ basis the spinors of equation (2.10) with the notations of equation (2.12) are given as

$$U(\mathbf{K}) = \begin{pmatrix} f(\boldsymbol{\sigma}\cdot\mathbf{K}) \\ (\boldsymbol{\sigma}\cdot\mathbf{K})g(\boldsymbol{\sigma}\cdot\mathbf{K}) \end{pmatrix},$$

$$V(-\mathbf{K}) = \begin{pmatrix} -(\boldsymbol{\sigma}\cdot\mathbf{K})g(\boldsymbol{\sigma}\cdot\mathbf{K}) \\ f(\boldsymbol{\sigma}\cdot\mathbf{K}) \end{pmatrix}. \tag{4.9}$$

This shows that the two component minimal coupling of equation (4.7) gets generated when $|vac'\rangle$ is gauge invariant. Also from equation (2.14) we have here

$$sin(2h_i(\boldsymbol{\sigma}\cdot\mathbf{K})) = cos(\chi_i(\boldsymbol{\sigma}\cdot\mathbf{K})) = e^{-\frac{1}{2}R_i^2(\boldsymbol{\sigma}\cdot\mathbf{K})^2}, \tag{4.10}$$

which will generate additional gauge couplings of quarks to photons.

The π^0 state of zero momentum is given as

$$|\pi^0(\mathbf{0})\rangle = \frac{1}{2\sqrt{3}}\int q_I(\mathbf{k})^\dagger \tau_3 \tilde{q}_I(-\mathbf{k})\tilde{u}_\pi(\mathbf{k})d\mathbf{k}|vac'\rangle. \tag{4.11}$$

In the above, $q_I^\dagger$ and $\tilde{q}_I$ are the two component isospin (u,d) doublet quark and antiquark creation operators with the spin, isospin and colour indices having been suppressed. Thus with minimal coupling the part of the Hamiltonian that gives rise to the process $\pi^0 \to 2\gamma$ is given as

$$H^{min} = \int \tilde{q}_I(-\mathbf{k}')^\dagger (V(-\mathbf{K})^\dagger(\boldsymbol{\alpha}\cdot\mathbf{K})U(\mathbf{K}))(\mathbf{k}',\mathbf{k})q_I(\mathbf{k})d\mathbf{k}d\mathbf{k}'. \tag{4.12}$$

We then obtain in the lowest order the S-matrix element for the corresponding transition as

$$\begin{aligned} &\langle \mathbf{k}_1,\lambda_1,\mathbf{k}_2,\lambda_2|S|\pi^0(\mathbf{0})\rangle \equiv \delta(P_f-P_i)M_{fi} \\ &= -i\times\frac{1}{2\sqrt{3}}\times 2\pi\delta(E_f-E_i)\times 3\times \\ &\int \langle \mathbf{k}_1,\lambda_1,\mathbf{k}_2,\lambda_2|\sum_{q=u,d} Tr\Big(V(-\mathbf{K}_q)^\dagger(\boldsymbol{\alpha}\cdot\mathbf{K}_q)\tau_3 U(\mathbf{K}_q)\Big)(\mathbf{k},\mathbf{k})|vac'\rangle\tilde{u}_\pi(\mathbf{k})d\mathbf{k}. \end{aligned} \tag{4.13}$$

We now note that from equation (4.9) we have the matrix equation,

$$\begin{aligned} &V(-\mathbf{K})^\dagger(\boldsymbol{\alpha}\cdot\mathbf{K})U(\mathbf{K}) \\ &= f(\boldsymbol{\sigma}\cdot\mathbf{K})(\boldsymbol{\sigma}\cdot\mathbf{K})f(\boldsymbol{\sigma}\cdot\mathbf{K}) - g(\boldsymbol{\sigma}\cdot\mathbf{K})(\boldsymbol{\sigma}\cdot\mathbf{K})^3 g(\boldsymbol{\sigma}\cdot\mathbf{K}) \\ &= \Big(2f(\boldsymbol{\sigma}\cdot\mathbf{K})^2-1\Big)(\boldsymbol{\sigma}\cdot\mathbf{K}) = e^{-\frac{1}{2}R^2(\boldsymbol{\sigma}\cdot\mathbf{K})^2}(\boldsymbol{\sigma}\cdot\mathbf{K}). \end{aligned} \tag{4.14}$$

In the last equation we have used [16] the identity $f(x)^2 + x^2 g(x)^2 = 1$. We note the special form of the above equation as being directly related to the vacuum structure through equation (2.14) or (4.10).

Clearly in equation (4.14) we have *any* number of photon fields, and, we are to choose only two of them to obtain the matrix element as in equation (4.13). We shall do so using that

$$\langle \mathbf{k}_1, \lambda_1 | \tilde{A}_i(\mathbf{k} - \mathbf{k}') | vac' \rangle = \frac{1}{\sqrt{2|\mathbf{k}_1|}} \delta(\mathbf{k}_1 + \mathbf{k} - \mathbf{k}') e_i(\mathbf{k}_1, \lambda_1).$$

For example, we first note that

$$\begin{aligned} &\langle \mathbf{k}_1, \lambda_1, \mathbf{k}_2, \lambda_2 | Tr[(\boldsymbol{\sigma} \cdot \mathbf{K}_q)^3(\mathbf{k}, \mathbf{k})] | vac' \rangle \\ &= \delta(\mathbf{k}_1 + \mathbf{k}_2) \times 2i\epsilon_{imj} \cdot \frac{e_q^2}{(2\pi)^3} \cdot \frac{(\mathbf{k}^m - \mathbf{k}_2^m) - (\mathbf{k}^m - \mathbf{k}_1^m)}{2|\mathbf{k}_1|} \times e_i(\mathbf{k}_1, \lambda_1) e_j(\mathbf{k}_2, \lambda_2) \times \frac{1}{\sqrt{2}}, \end{aligned} \tag{4.15}$$

and that

$$\begin{aligned} &\langle \mathbf{k}_1, \lambda_1, \mathbf{k}_2, \lambda_2 | Tr[(\boldsymbol{\sigma} \cdot \mathbf{K}_q)^5(\mathbf{k}, \mathbf{k})] | vac' \rangle \\ &= \delta(\mathbf{k}_1 + \mathbf{k}_2) \times 2i\epsilon_{imj} \cdot \frac{e_q^2}{(2\pi)^3} \cdot \frac{2\mathbf{k}_1^m}{2|\mathbf{k}_1|} \times e_i(\mathbf{k}_1, \lambda_1) e_j(\mathbf{k}_2, \lambda_2) \times \frac{1}{\sqrt{2}} . (3k^2 + q^2), \end{aligned} \tag{4.16}$$

where, $k^2 = \mathbf{k}^2$, and, $q^2 = (\mathbf{k} - \mathbf{k}_1)^2$, and we are using that $\tilde{u}_\pi(\mathbf{k})$ is even in $\mathbf{k}$. Proceeding in a similar manner, from equation (4.13) we then obtain that

$$M_{fi} = \frac{\alpha}{\pi} \cdot \frac{1}{2\sqrt{3}} \cdot \epsilon_{imj} \cdot \frac{\mathbf{k}_1^m}{|\mathbf{k}|} e_i(\mathbf{k}_1, \lambda_1) e_j(\mathbf{k}_2, \lambda_2) \cdot R^2 I(R, m_\pi), \tag{4.17}$$

where I is given as, with $|\mathbf{k}_1| = m_\pi/2$,

$$\begin{aligned} I(R, m_\pi) = \int \tilde{u}_\pi(\mathbf{k}) d\mathbf{k} \Big[1 &- \frac{1}{2!}\left(\frac{R^2}{2}\right)(3k^2 + q^2) + \frac{1}{3!}\left(\frac{R^2}{2}\right)^2 (5k^4 + 3k^2q^2 + q^4) \\ &- \frac{1}{4!}\left(\frac{R^2}{2}\right)^3 (7k^6 + 5k^4q^2 + 3k^2q^4 + q^6) \\ &+ \frac{1}{5!}\left(\frac{R^2}{2}\right)^4 (9k^8 + 7k^6q^2 + 5k^4q^4 + 3k^2q^6 + q^8) + \cdots \Big]. \end{aligned} \tag{4.18}$$

In the above we have used that $3(e_u^2 - e_d^2) = e^2$ as indicating separately the contributions from u and d quarks. We know the wave function $\tilde{u}_\pi(\mathbf{k})$, and hence calculating the above integral we obtain that $I = 0.0858$ GeV$^{3/2}$. The decay width then becomes

$$\Gamma(\pi^0 \to 2\gamma) = \frac{m_\pi^2}{4} \cdot \sum_{\lambda_1\lambda_2} |M_{fi}|^2 = m_\pi^2 \cdot \frac{\alpha^2}{48\pi^2} \cdot R^4 I^2 \approx 8.3 \text{ eV}. \tag{4.19}$$

This may be compared with the experimental value given as 7.7±0.6 eV. The agreement here *without* fixing any parameter thus appears to be rather good. The anomaly equation has not been used, and the vacuum structure as determined through f_π seems to be adequate.

It thus appears that the two component minimal coupling contains within itself the effects of anomaly! Instead of the anomaly equation, gauge invariance while considering restructuring of vacuum does the job. This may have consequences beyond the verification of the above width for the π^0 decay. Here as noted gauge invariance at the two component level *gets generated* since the spinors are related to the vacuum structure. The equation (4.19) in the context of chiral symmetry restoration in quark gluon plasma is further discussed in section **VII**.

V. MASSES OF MESONS

In the last section we have taken the wave function of the pion as given through chiral symmetry breaking. Simultaneously, in the expressions involving the pion mass, we have naturally substituted the mass of the pion as observed. However, we know that the pion is only *approximately* a Goldstone boson, and that m_u, m_d do not vanish. We shall now include this effect to relate Lagrangian masses with the masses of the mesons. We shall however avoid the diagonal pseudoscalar mesons, where anomaly effects from the gluonic contributions will be there [14], which are not being considered in the present paper.

We now take the mass part of the Hamiltonian density as

$$\mathcal{H}_{mass}(\mathbf{x}) = \sum m_i \bar{\psi}_i(\mathbf{x})\psi_i(\mathbf{x}). \tag{5.1}$$

As before, i is the flavour, and summation over colour is understood. From the definition of the pion state as obtained through chiral symmetry breaking in equation (3.2), we then obtain for example the mass of π^+ as, using translational invariance,

$$
\begin{aligned}
m(\pi^+) &= (2\pi)^3 \cdot \frac{N_\pi^2}{6} \langle vac'|Q_{\pi^+}^\dagger \mathcal{H}_{mass}(\mathbf{x}) Q_{\pi^+}|vac'\rangle \\
&\equiv (2\pi)^3 \cdot \frac{N_\pi^2}{6} \cdot \frac{1}{2} \cdot \langle vac'|[[Q_{\pi^+}^\dagger, \mathcal{H}_{mass}(\mathbf{x})], Q_{\pi^+}]|vac'\rangle \\
&= (2\pi)^3 \cdot \frac{N_\pi^2}{6} \cdot \frac{1}{2} \cdot (m_1 + m_2)\langle vac'|[-\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2]|vac'\rangle. \qquad (5.2)
\end{aligned}
$$

From the above equation, using the earlier expression for f_π as in equation (3.9) we can obtain the conventional current algebra result that

$$
m_\pi^2 = \frac{m_u + m_d}{2} \cdot \frac{-\langle \bar{u}u\rangle - \langle \bar{d}d\rangle}{f_\pi^2}. \qquad (5.3)
$$

However, with the present ansatz we also have a simpler relationship when $R_1 = R_2$ with the pion mass expressed in terms of the current quark masses. This is given from equations (3.21) and (5.2) as

$$
m_\pi = \frac{m_u + m_d}{2} \cdot 4\sqrt{2}, \qquad (5.4)
$$

where, the explicit dependance on R cancels out. Equation (5.4) yields that

$$
\frac{m_u + m_d}{2} \approx 24 \text{ MeV}. \qquad (5.5)
$$

We note that the current quark masses for u and d quarks are rather large, and, here the pion mass becomes independent of the condensate scale.

In the same way we can also get the current quark mass m_s. For this purpose we note that, with N_K given by equation (3.15),

$$
\begin{aligned}
m(K^+) &= (2\pi)^3 \cdot \frac{N_K^2}{6} \cdot \frac{1}{2}(m_1 + m_3)\langle vac'|[-\bar{\psi}_1\psi_1 - \bar{\psi}_3\psi_3]|vac'\rangle \\
&= (2\pi)^3 \cdot \frac{N_K^2}{6} \cdot \frac{1}{2}(m_1 + m_3) \times \frac{3}{\sqrt{2\pi}\sqrt{\pi}} \times \left(\frac{1}{R_1^3} + \frac{1}{R_3^3}\right). \qquad (5.6)
\end{aligned}
$$

This yields that

$$
m_3 = m_s \approx 96 \text{ MeV}. \qquad (5.7)
$$

The current quark mass for the strange quark here appears to be smaller than usual. From equation (3.18) we may also obtain the conventional current algebra result that

$$m_K^2 = \frac{m_u + m_s}{2} \cdot \frac{-\langle \bar{u}u \rangle - \langle \bar{s}s \rangle}{f_K^2}. \tag{5.8}$$

We may compare the *explicit* symmetry breaking parameters m_i with the vacuum re-alignment scales $1/R_i$. We then note that for example

$$\frac{m_u + m_d}{2} \cdot R_1 = 0.118, \qquad \text{and} \qquad m_s \cdot R_s = .194. \tag{5.9}$$

Hence when we determine the wave function from equations like (3.7) with breaking of exact chiral symmetry, we expect the K-meson wave function to be less reliable than for the pion. As a curiousity, *if* there were a pseudoscalar meson with only strange quarks, parallel to equation (5.4) we shall have

$$m(\bar{s}s) = m_s \times 4\sqrt{2} = 543 \text{ MeV}. \tag{5.10}$$

We note that the isospin diagonal mass matrix shall in addition have gluonic anomaly contributions [14] and equation (5.10) does not include the same. There will really be a mass matrix connecting to other states. We shall not deal with the same here.

We will however still adopt the same philosophy for the heavier nondiagonal mesons as a 'zeroeth order' approximation. For these mesons, the corresponding decay constants are not known. Hence we adopt a different strategy. We note that for each quark here we are introducing only two parameters. We first introduce an assumed vacuum structure given by the parameter R_i, and then introduce the Lagrangian quark masses m_i to obtain the finite masses of the mesons. For the D-mesons we know that $m(D^+) = 1869$ MeV, and that $m(D_s)$ =1968 MeV. Let us now try to determine the values of $R_4 = R_c$ and $m_4 = m_c$. We note that as earlier the D^+ state is given as

$$\begin{aligned} \mid D^+(\mathbf{0}) >= N_D \cdot \frac{1}{\sqrt{6}} \int c_I(\mathbf{k})^\dagger \tilde{d}_I(-\mathbf{k})[cos(\frac{1}{2}\chi_2(\mathbf{k}))cos(\frac{1}{2}\chi_4(\mathbf{k})) \\ -sin(\frac{1}{2}\chi_2(\mathbf{k}))sin(\frac{1}{2}\chi_4(\mathbf{k}))]d\mathbf{k} \mid vac' >, \end{aligned} \tag{5.11}$$

where the normalisation constant N_D can be calculated. The D-meson mass is now given as

$$m(D^+) = (2\pi)^3 \cdot \frac{N_D^2}{6} \cdot \frac{1}{2}(m_2 + m_4)\langle vac'|[-\bar{\psi}_2\psi_2 - \bar{\psi}_4\psi_4]|vac'\rangle$$
$$= \frac{N_D^2}{6} \cdot \frac{1}{2}(m_d + m_c) \times \frac{3\pi^{3/2}}{\sqrt{2}} \times \left(\frac{1}{R_2^3} + \frac{1}{R_4^3}\right). \tag{5.12}$$

Similarly, the D_s mass is given as

$$m(D_s^+) = \frac{N_{D_s}^2}{6} \cdot \frac{1}{2}(m_s + m_c) \times \frac{3\pi^{3/2}}{\sqrt{2}} \times \left(\frac{1}{R_s^3} + \frac{1}{R_c^3}\right). \tag{5.13}$$

We note that parallel to equation (3.15) the normalisations N_D and N_{D_s} are given through

$$\frac{1}{N_D^2} = 2\pi \int \Big[1 - sin(\chi_2(|\mathbf{k}|))sin(\chi_4(|\mathbf{k}|)) + cos(\chi_2(|\mathbf{k}|))cos(\chi_4(|\mathbf{k}|))\Big]|\mathbf{k}|^2 d|\mathbf{k}|, \tag{5.14}$$

and,

$$\frac{1}{N_{D_s}^2} = 2\pi \int \Big[1 - sin(\chi_3(|\mathbf{k}|))sin(\chi_4(|\mathbf{k}|)) + cos(\chi_3(|\mathbf{k}|))cos(\chi_4(|\mathbf{k}|))\Big]|\mathbf{k}|^2 d|\mathbf{k}|. \tag{5.15}$$

We now use equations (5.12) and (5.13) along with the appropriate masses for the particles to solve for the new parameters m_c and R_c numerically. We then obtain that

$$m_4 = m_c \approx 355 \text{ MeV}; \qquad R_4^2 = R_c^2 = 0.4852 \text{ GeV}^{-2}. \tag{5.16}$$

We also note that here

$$m_c \cdot R_c = 0.247. \tag{5.17}$$

As stated earlier, this can be regarded as a measure of error in the wave function obtained from exact chiral symmetry breaking. We also predict the corresponding decay constants to be

$$f_D = 248 \text{ MeV}; \qquad f_{D_s} = 261 \text{ MeV}. \tag{5.18}$$

We use the same strategy for the B-mesons. We note that $m_B = 5.277$ GeV, and that $m_{B_s} = 5.383$ GeV. As earlier we take m_b and R_b as unknown, and then determine the same from the known values of the above masses. This yields that

$$m_b \approx 1.04 \text{ GeV}; \qquad R_b^2 = 0.2307 \text{ GeV}^{-2}. \tag{5.19}$$

Here $m_b R_b = 0.50$, which is rather large. We also obtain that $m_{B_c} = 5.586$ GeV, and, the corresponding decay constants as

$$f_B = 256 \text{ MeV}; \qquad f_{B_s} = 261 \text{ MeV}; \qquad f_{B_c} = 475 \text{ MeV}. \tag{5.20}$$

We note that in the above as the current quark mass increases, the product $m_i R_i$ becomes larger and larger. Hence to describe the meson wave functions through only the chiral symmetry breaking mechanism becomes less and less valid. For heavy mesons, it is only a 'zeroeth order' approximation, but the surprising feature is that always the masses from chiral symmetry breaking dominate over the current quark masses. Hence with the present hypothesis the heavy quarks can not be approximated by Dirac spinors. This shall have an effect on spectroscopic properties for the spin dependance of the energy levels for heavy hadrons.

VI. PION AND KAON CHARGE RADII

We shall here briefly recall the calculation of the pion charge radius [9] and then obtain the same for the kaon. The pion form factor is given by the equation [16]

$$< \pi^+(-\mathbf{p})|J^0(0)|\pi^+(\mathbf{p}) >= \frac{1}{(2\pi)^3} \cdot \frac{m_\pi}{p^0} \cdot G_E^\pi(t) \tag{6.1}$$

where through direct evaluation [16] we obtained that

$$G_E^\pi(t) = \sum_q e_q \int \tilde{u}_\pi(\mathbf{k}_1')^\dagger \tilde{u}_\pi(\mathbf{k}_1) \left(f_q(\mathbf{k}_1') f_q(\mathbf{k}_1) + \mathbf{k}_1' \cdot \mathbf{k}_1 g_q(\mathbf{k}_1') g_q(\mathbf{k}_1) \right) d\mathbf{k}_1. \tag{6.2}$$

In the above, q stands for u or d quark, and, [16],

$$t = -4\mathbf{p}^2; \qquad \mathbf{k}_1' = \mathbf{k}_1 - \frac{m_\pi}{p^0}\mathbf{p}. \tag{6.3}$$

For the charge radius we retain terms only upto $|\mathbf{p}|^2$ and identify the same through the equation

$$G_E(t) = 1 + \frac{R_{ch}^2}{6}\, t\ . \tag{6.4}$$

We now substitute $\mathbf{k}_1' = \mathbf{k} - \frac{1}{2}\mathbf{p}$ and $\mathbf{k}_1 = \mathbf{k} + \frac{1}{2}\mathbf{p}$. Upto order $|\mathbf{p}|^2$, along with other equations we also have [9]

$$\hat{k}_1' \cdot \hat{k}_1 = 1 - \frac{\mathbf{p}^2}{3\mathbf{k}^2}. \tag{6.5}$$

On simplification, we had finally obtained [9]

$$R_{ch}^2 = R_{ch1}^2 + R_{ch2}^2 \tag{6.6}$$

where,

$$R_{ch1}^2 = \frac{N_\pi^2}{4} \int (\bigtriangledown cos\chi(\mathbf{k}))^2 d\mathbf{k} \tag{6.7}$$

is the contribution coming from the wave function alone, and,

$$R_{ch2}^2 = \frac{N_\pi^2}{16} \int cos^2\chi(\mathbf{k}) \left[\frac{R_1^4 \mathbf{k}^2 cos^2\chi(\mathbf{k})}{1 - cos^2\chi(\mathbf{k})} + \frac{4(1 - cos\chi(\mathbf{k}))}{\mathbf{k}^2} \right] d\mathbf{k} \tag{6.8}$$

is the balance of the contribution. In the second term of the right hand side above we have corrected [9] a factor two.

For the choice of equation (3.4) and the expression for $cos\chi(\mathbf{k}) = cos\chi_1(\mathbf{k})$, we then obtain that $R_{ch1}^2 = 8.842$ GeV^{-2}, and $R_{ch2}^2 = 2.591$ GeV^{-2}. We thus obtain,

$$R_{ch} = 0.67 \text{ fms}, \tag{6.9}$$

which may be compared with the experimental value of $R_{ch} = 0.66$ fms [20]. The result is quite good as compared to the last calculation [9] on correcting the error.

For the K-meson, finding out the charge radius is more complicated. The form factor to be considered here is given through

$$< K^+(-\mathbf{p})|J^0(0)|K^+(\mathbf{p}) >= \frac{1}{(2\pi)^3} \frac{m_K}{p^0} G_E^K(t) \tag{6.10}$$

Parallel to equation (6.2), with q quark as the interacting quark, we have

$$G_E^K(t) = \sum_q e_q \int \tilde{u}_K(\mathbf{k}_1')^\dagger \tilde{u}_K(\mathbf{k}_1) \left(f_q(\mathbf{k}_1') f_q(\mathbf{k}_1) + \mathbf{k}_1' \cdot \mathbf{k}_1 g_q(\mathbf{k}_1') g_q(\mathbf{k}_1) \right) d\mathbf{k}_1, \tag{6.11}$$

for u and s quark with momenta as below. The state of finite momentum is constructed here with Lorentz boosting [16], where we need the energy shared by the quark and antiquark at rest to obtain the time dependance of the operators for boosting. A specific threshold enhancement arising from this feature has been noted earlier [21]. In the pion both quark and antiquark equally share the pion rest energy which gives the energy shared as half the pion mass. However, kaon being an asymmetric system, the energy shared by the quarks shall be different, and the time dependance is unknown [16]. This gives an ambiguity for the determination of the charge radius.

Let k_1 and k_2 be the *four-momenta* of the u and $\bar{s}$ for K^+ at rest [16] where $k_1^0 = \lambda_1 m_K$ and $k_2^0 = \lambda_2 m_K$ with $\lambda_1 + \lambda_2 = 1$. For the state $|K^+(\mathbf{p})\rangle$, the above momenta will be transformed through the Lorentz matrix $L(p)$ taken as, with $\mu = 0, \cdots, 3$ and $i, j = 1, 2, 3$ [16]

$$L(p)_{\mu 0} = L(p)_{0\mu} = \frac{p^\mu}{m_K} \quad \text{and} \quad L(p)_{ij} = \left(\delta_{ij} + \frac{p^i p^j}{m_K(p^0 + m_K)}\right). \tag{6.12}$$

In the above $p^0 = (\mathbf{p}^2 + m_K^2)^{1/2}$. Similarly for the state $|K^+(-\mathbf{p})\rangle$ let the quark momenta at rest be k_1' and k_2' which are Lorentz boosted by matrix $L(p')$ with $\mathbf{p'} = -\mathbf{p}$. We note that we have also $k_1'^0 = \lambda_1 m_K$ and $k_2'^0 = \lambda_2 m_K$. Let $G_E^{K1}(t)$ be the contribution for form factor where u-quark interacts. Since here $\bar{s}$ is spectator, we get momentum conservation equation as [16]

$$L(p)_{ij} k_2^j + L(p)_{i0} k_2^0 = L(p')_{ij} k_2'^j + L(p')_{i0} k_2^0. \tag{6.13}$$

On multiplyng the above by the inverse of the 3×3 matrix $L(p)_{ij} = L(p')_{ij}$, we then obtain that

$$\mathbf{k}_2' = \mathbf{k}_2 + \lambda_2 \frac{m_K}{p^0} \cdot 2\mathbf{p}, \qquad \text{or,} \qquad \mathbf{k}_1' = \mathbf{k}_1 - \lambda_2 \frac{m_K}{p^0} \cdot 2\mathbf{p}. \tag{6.14}$$

We then replace $\mathbf{k}_1$ by the symmetric integration variable $\mathbf{k}$ with the substitutions

$$\mathbf{k}_1' = \mathbf{k} - \lambda_2 \frac{m_K}{p^0} \mathbf{p}; \qquad \mathbf{k}_1 = \mathbf{k} + \lambda_2 \frac{m_K}{p^0} \mathbf{p}. \tag{6.15}$$

Thus, when the u-quark interacts we obtain that

$$G_E^{K1}(t) = e_u \int \tilde{u}_K(\mathbf{k}_1')^\dagger \tilde{u}_K(\mathbf{k}_1) \left(f_1(\mathbf{k}_1') f_1(\mathbf{k}_1) + \mathbf{k}_1' \cdot \mathbf{k}_1 g_1(\mathbf{k}_1') g_1(\mathbf{k}_1) \right) d\mathbf{k}, \tag{6.16}$$

where the variables are as given in equation (6.15). We also note that spin rotations have been included [16], and, there is no contribution from the same as we have here $S(L(p'))^\dagger S(L(p)) = I$ for $\mathbf{p}' = -\mathbf{p}$.

There will be a parallel contribution $G_E^{K2}(t)$ where the $\bar{s}$ interacts and u is the spectator. This contribution is obtained in a similar manner as

$$G_E^{K2}(t) = -e_s \int \tilde{u}_K(\mathbf{k}_1')^\dagger \tilde{u}_K(\mathbf{k}_1) \left(f_3(\mathbf{k}_1') f_3(\mathbf{k}_1) + \mathbf{k}_1' \cdot \mathbf{k}_1 g_3(\mathbf{k}_1') g_3(\mathbf{k}_1) \right) d\mathbf{k}, \tag{6.17}$$

where $\lambda_2 \to \lambda_1$, and $e_u \to -e_s$. Here parallel to equation (6.15) we have $\mathbf{k}_1' = \mathbf{k} - \lambda_1 \frac{m_K}{p^0} \cdot \mathbf{p}$, and, $\mathbf{k}_1 = \mathbf{k} + \lambda_1 \frac{m_K}{p^0} \cdot \mathbf{p}$. On simplification we then obtain that

$$R_{chK}^2 = R_{chK1}^2 + R_{chK2}^2 \tag{6.18}$$

where, parallel to equation (6.7)

$$R_{chK1}^2 = \left(\frac{2}{3}\lambda_2^2 + \frac{1}{3}\lambda_1^2 \right) \times \int (\bigtriangledown \tilde{u}_K(\mathbf{k}))^2 d\mathbf{k} \tag{6.19}$$

is the contribution coming from the wave function of equation (3.14), and,

$$\begin{aligned} R_{chK2}^2 = {} & \frac{2}{3} \times \lambda_2^2 \int u_K(\mathbf{k})^2 \left\{ \frac{R_1^4 \mathbf{k}^2 cos^2(\chi_1(\mathbf{k}))}{4(1 - cos^2\chi_1(\mathbf{k}))} + \frac{(1 - cos\chi_1(\mathbf{k}))}{\mathbf{k}^2} \right\} d\mathbf{k} \\ & + \frac{1}{3} \times \lambda_1^2 \int u_K(\mathbf{k})^2 \left\{ \frac{R_3^4 \mathbf{k}^2 cos^2(\chi_3(\mathbf{k}))}{4(1 - cos^2\chi_3(\mathbf{k}))} + \frac{(1 - cos\chi_3(\mathbf{k}))}{\mathbf{k}^2} \right\} d\mathbf{k}. \end{aligned} \tag{6.20}$$

is the balance of the contribution. We may easily note that when $R_1 = R_3$ and $\lambda_1 = \lambda_2 = \frac{1}{2}$, the above expressions go over to the corresponding expressions for the pion. The first term in the curly brackets above came from the simplification

$$\begin{aligned} & cos\left(\frac{1}{2}\chi_1(\mathbf{k}_1')\right) cos\left(\frac{1}{2}\chi_1(\mathbf{k}_1)\right) + sin\left(\frac{1}{2}\chi_1(\mathbf{k}_1')\right) sin\left(\frac{1}{2}\chi_1(\mathbf{k}_1)\right) \\ \approx {} & 1 - \frac{R_1^4 \mathbf{k}^2 cos^2\chi_1(\mathbf{k})}{6(1 - cos^2\chi_1(\mathbf{k}))} \times \lambda_2^2 \mathbf{p}^2, \end{aligned} \tag{6.21}$$

and, the second term came from

$$sin^2\left(\frac{1}{2}\chi_1(\mathbf{k})\right)\hat{\mathbf{k}}'_1\cdot\hat{\mathbf{k}}_1 \rightarrow -\frac{2(1-cos\chi_1(\mathbf{k}))}{3\mathbf{k}^2}\times\lambda_2^2\mathbf{p}^2. \tag{6.22}$$

The above contributions are for $\bar{s}$ being the spectator, and, we have used equations (2.14) and (6.15) for the simplification. The second curly bracket arises on interchanging the two quarks.

We now have to estimate λ_1 and λ_2. We had earlier suggested [22] that for sharing of the energy at rest, the kinetic energies of the two constituents may be different, but the potential energy shall be equally shared. In the present determination of the mass of the kaon the potential picture is absent. We shall however extrapolate the same by looking at the expression in (5.6) to guess these factors. We shall consider here two possible identifications. From equation (5.6) let us "identify" the potential energy as

$$v_K = (2\pi)^3\cdot\frac{N_K^2}{6}\cdot\frac{1}{2}[m_1\langle vac'|-\bar{\psi}_3\psi_3|vac'\rangle + m_3\langle vac'|-\bar{\psi}_1\psi_1|vac'\rangle]. \tag{6.23}$$

The balance of the contributions contain only u terms or only s terms, which we identify as the respective kinetic contributions. We then obtain that [22] $\lambda_1 = 0.134$ and $\lambda_2 = 0.866$. This yields that $R^2_{chK1} = 4.39$ GeV^{-2} and $R^2_{chK2} = 1.20$ GeV^{-2}, so that

$$R_{chK} = 0.47 \text{ fms}. \tag{6.24}$$

We may otherwise identify that in equation (5.6) the $\langle\bar{u}u\rangle$ part corresponds to $\lambda_1 m_K$, and, the $\langle\bar{s}s\rangle$ part corresponds to $\lambda_2 m_K$. We then have $\lambda_1 = 0.067$ and $\lambda_2 = 0.933$. This yields that $R^2_{chK1} = 5.04$ GeV^{-2} and $R^2_{chK2} = 1.37$ GeV^{-2}, so that

$$R_{chK} = 0.50 \text{ fms}. \tag{6.25}$$

The above values may be compared with the experimental value of $R_{chK} = 0.58$ fms [14]. The calculated value appears to be small, and indicates that taking the wave function as determined from *exact* chiral symmetry breaking may not be correct. The "identification" of the fractions λ_1, λ_2 is also unreliable. We *should* have a handle on spectroscopy which may clarify the above as well as give corrections to the kaon wave function as different from the purely vacuum structure contribution as in equation (3.14).

VII. DISCUSSIONS

Let us recall what has been achieved here. We first neglect masses of the quarks and assume that global chiral symmetry breaks spontaneously. This is described through a vacuum realignment where we approximate for the correlation of a quark at two different space points as in equations (2.13) and (2.14) with a Gaussian function. Thus for the vacuum structure for quark q_i, we introduce a single parameter R_i. This parameter also gives the four component quark field operators for exact chiral symmetry breaking. We then find that R_i can be determined from the experimental value of the decay constant. Further, for approximate chiral symmetry breaking, we obtain the masses of the mesons through current algebra as in equations like (5.6) or (5.8) in terms of the Lagrangian masses of the quarks. This introduces another constant m_i for each quark. With only these two parameters for each quark, we use the explicit form of vacuum realignment to draw conclusions for the hadronic properties and examine consistency of such a hypothesis.

For the light quark sector, we find that the parameter R for the vacuum structure as determined from f_π also yields $\Gamma(\pi^0 \to 2\gamma)$ and R_{ch}^2 correctly. For the π^0 decay, we use the fact that the destabilised vacuum $|vac'\rangle$ should remain gauge invariant. For the charge radius, we use that mesons in motion should be obtained through Lorentz boosting [16]. It thus appears that *we know* the vacuum structure for quark condensates of the light quark sector from experimental observations of the above hadronic properties as conjectured earlier [9].

We also determine the vacuum structure of the s-quark sector from the experimental value of f_K, and, using the same, go on to derive the charge radius of the kaon. This falls short of the experimental value by about fifteen percent. We may recall that from chiral perturbation theory a similar disagreement is also there, where the theoretical value is larger by about the same amount [14].

We have next calculated the decay constants of D and B mesons. In this sector a quantitative agreement of the same is not expected. We find however that with the present

hypothesis, the spinor structure for c and b quarks may be quite different from that of a free Dirac particle. This feature shall have consequences for spectroscopy of heavy mesons.

Let us now note some obvious limitations of the present calculations. The s-quark seems to have a Lagrangian mass of the order of 100 MeV, and the corresponding masses of c and b quark are higher. For them we have considered the wave functions of the mesons as obtained totally from chiral symmetry breaking. In some sense this may not be very bad as *post facto* the scales for chiral symmetry breaking for them are higher than the above masses. It is however desirable to look further into this. A related assumption has been that for all the quarks we have taken $\chi_0(\mathbf{k}) = \frac{\pi}{2}$. On the other hand, as per equation (2.6) we could have taken some thing like $cos\chi_0(\mathbf{k}) = m_i/\epsilon_0(\mathbf{k})$ and $sin\chi_0(\mathbf{k}) = |\mathbf{k}|/\epsilon_0(\mathbf{k})$ with $\epsilon_0(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m_i^2}$, which corresponds to free fields for $m_i \neq 0$. We have not done the same here since our main objective has been to relate the vacuum structure for chiral symmetry breaking to hadronic properties with simple calculations, and show that this structure is observable. For light quarks the programme works unexpectedly well. For heavy quarks, we believe that the refinements mentioned above shall be inevitable.

As $R_i \to \infty$, chiral symmetry gets restored. We note from equation (5.4) that the π^0 mass will remain unaltered. From equation (4.19) however with a direct evaluation we note that then $\Gamma(\pi^0 \to 2\gamma)$ continuously increases as R^2 increases. In particular, with R^2 respectively as 30 GeV^{-2}, 50 GeV^{-2} and 70 GeV^{-2} for the vacuum structure, the corresponding values of $\Gamma(\pi^0 \to 2\gamma)$ are given as 9.2 eV, 11.3 eV and 12.7 eV. This may be relevant for observing chiral symmetry restoration in quark gluon plasma (QGP) during an intermediate phase when quarks and gluons and hadrons are present. Since with temperature R is likely to increase, this will show up in $\Gamma(\pi^0 \to 2\gamma)$ over its value at zero temperature and may be looked for as a signal for progress towards chiral symmetry restoration prior to phase transition. We note that J/ψ suppression [23] has been a conventional signature for QGP [24]. Can we also calculate and expect a variation of the η, η' and η_c widths? For this purpose it shall be necessary to see whether anomaly effects of the gluonic sector [14] can be absorbed through a mechanism similar to what has been done for the electromagnetic

sector for π^0 decay. The real problem is to develop a technology parallel of conventional spectroscopy, which includes in an essential manner approximate chiral symmetry breaking.

Intuitively one might expect that π^0 dissolves more easily as R increases [23]. The calculation here seems to illustrate the same. Also, most of the π^0 decay will take place during hadronisation process or later, when the temperature will be less. From equations (3.5) and (3.9) there will also be a similar dependance of f_π on R^2. In contrast to π^0, as R^2 increases $\pi^\pm$ progressively become more stable for the leptonic decay mode.

The present work generalises Ref. [9] in being able to obtain the π^0 width through a derived form of minimal coupling. It also gives a theoretical base for the *ad hoc* assumptions of Ref. [16], applied to many coherent and incoherent processes [25,26]. It may be desirable to see how these results change with the present form of equation (2.12) related to vacuum structure of (2.14). The bright side here is that we now seem to know the vacuum structure for light quarks. However, it also emphasizes the limitations in our understanding the same for heavier quarks, while illustrating their relevance for the corresponding hadronic properties.

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